ST HILDA'S
ANGLICAN SCHOOL FOR GIRLS INC.

Total Time: 25 minutes

Total Marks: 19 marks

Specialist Mathematics Units 3&4

Topic Test 2
(Wed, May 11th)

Resource Free

ClassPad Calculators are NOT permitted.
Miscellaneous Formulae Sheet is permitted.

Name: ANSWERS

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1. [1 & 2 = 3 marks]

Points A, B and C are such that $\overrightarrow{AB} = 2\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}$ and $\overrightarrow{AC} = -\underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k}$.

(a) Find a vector that is perpendicular to vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

$\overrightarrow{AB}$: 2 1 −1

$\overrightarrow{AC}$: −1 2 2

$\underset{\sim}{n} = 4\underset{\sim}{i} - 3\underset{\sim}{j} + 5\underset{\sim}{k}$ (1)

(b) Point A has position vector $3\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}$.

(i) Find the vector equation of the plane that contains points A, B and C.

$$\underset{\sim}{r} \cdot \begin{pmatrix} 4 \\ -3 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -3 \\ 5 \end{pmatrix}$$

$$= -1 \quad (1)$$

or $\underset{\sim}{r} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$

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(ii) Find the position vector of point B.

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$

$$\overrightarrow{OB} = \overrightarrow{AB} + \overrightarrow{OA}$$

$$= \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} 5 \\ 2 \\ -3 \end{pmatrix} \quad (1)$$

3

2. **[1, 1, 1, 1 & 2 = 6 marks]**

Consider the two points, *A* and B, with position vectors $\underset{\sim}{a} = \underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} - 3\underset{\sim}{j} + 4\underset{\sim}{k}$.

(a) Find $2\underset{\sim}{a} + \underset{\sim}{b}$

$$2\begin{pmatrix}1\\2\\2\end{pmatrix} + \begin{pmatrix}2\\-3\\4\end{pmatrix} = \begin{pmatrix}4\\1\\8\end{pmatrix} \quad (1)$$

(b) Find the magnitude of $\underset{\sim}{a}$.

$$|\underset{\sim}{a}| = \sqrt{1^2 + 2^2 + 2^2}$$
$$= 3 \quad (1)$$

(c) Vector $\underset{\sim}{f}$ is in the direction of $\underset{\sim}{a}$ and has magnitude of 5. Find vector $\underset{\sim}{f}$.

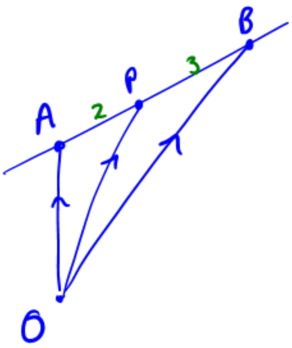
$$\underset{\sim}{f} = \frac{1}{3}\underset{\sim}{a} \times 5$$
$$= \frac{5}{3}\begin{pmatrix}1\\2\\2\end{pmatrix} \quad (1)$$

6

(d) Find the vector $\overrightarrow{AB}$.

$$\overrightarrow{AB} = \begin{pmatrix}2\\-3\\4\end{pmatrix} - \begin{pmatrix}1\\2\\2\end{pmatrix}$$
$$= \begin{pmatrix}1\\-5\\2\end{pmatrix} \quad (1)$$

(e) Find the position vector of the point *P* that divides *AB* internally in the ratio of 2:3.
i.e. AP:PB = 2:3.



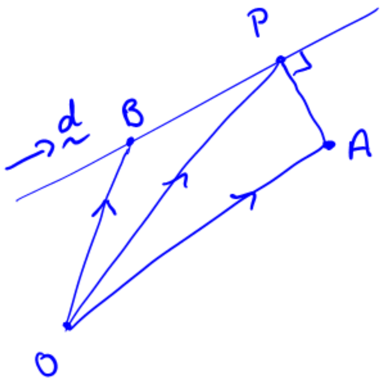
$$\overrightarrow{OP} = \overrightarrow{OA} + \frac{2}{5}\overrightarrow{AB} \quad (1)$$
$$= \begin{pmatrix}1\\2\\2\end{pmatrix} + \frac{2}{5}\begin{pmatrix}1\\-5\\2\end{pmatrix}$$
$$= \begin{pmatrix}1\frac{2}{5}\\0\\2\frac{4}{5}\end{pmatrix} \text{ or } \begin{pmatrix}1.4\\0\\2.8\end{pmatrix} \text{ or } \frac{1}{5}\begin{pmatrix}7\\0\\14\end{pmatrix}$$
(1)

6

3. **[4 marks]**

Find the exact shortest distance between the line with vector equation $\underset{\sim}{r}(\lambda)=\begin{pmatrix}\lambda \\ 3+2\lambda\end{pmatrix}$ and the point A with position vector $\underset{\sim}{i}+10\underset{\sim}{j}$.

Let P be point on line closest to A where $\lambda=\lambda_1$.



$\underset{\sim}{d}=\begin{pmatrix}1\\2\end{pmatrix}$ $\quad \vec{OB}=\begin{pmatrix}0\\3\end{pmatrix}$

$\vec{OP}=\underset{\sim}{r}(\lambda_1)=\begin{pmatrix}\lambda_1\\3+2\lambda_1\end{pmatrix}$

$\vec{PA}=\vec{OA}-\vec{OP}=\begin{pmatrix}1\\10\end{pmatrix}-\begin{pmatrix}\lambda_1\\3+2\lambda_1\end{pmatrix}=\begin{pmatrix}1-\lambda_1\\7-2\lambda_1\end{pmatrix}$ (1)

As $\vec{PA}$ is $\perp$ to line,

$$\vec{PA}\cdot\underset{\sim}{d}=0$$

$$\begin{pmatrix}1-\lambda_1\\7-2\lambda_1\end{pmatrix}\cdot\begin{pmatrix}1\\2\end{pmatrix}=0 \quad (1)$$

$$1-\lambda_1+2(7-2\lambda_1)=0$$

$$15-5\lambda_1=0$$

$$\lambda_1=3 \quad (1)$$

$$\therefore \vec{PA}=\begin{pmatrix}1-\lambda_1\\7-2\lambda_1\end{pmatrix}\Bigg|_{\lambda_1=3}=\begin{pmatrix}-2\\1\end{pmatrix}$$

$$|\vec{PA}|=\sqrt{5}$$

So, shortest distance from line to A is $\sqrt{5}$. (1)

$\frac{}{4}$

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4. [1, 2 & 3 = 6 marks]

Two particles are moving on paths described by the vector equations $\underset{\sim}{\mathbf{r}}_A = (3t-1)\underset{\sim}{\mathbf{i}} + 5t\underset{\sim}{\mathbf{j}}$ and $\underset{\sim}{\mathbf{r}}_B = (2t+5)\underset{\sim}{\mathbf{i}} + (t^2-6)\underset{\sim}{\mathbf{j}}$ respectively.

(a) Find the Cartesian equation of the path of particle B.

For B: $x = 2t+5 \Rightarrow t = \frac{x-5}{2}$

$y = t^2 - 6$

$\therefore \quad y = \left(\frac{x-5}{2}\right)^2 - 6 \quad (1)$

$= \frac{1}{4}(x-5)^2 - 6$

(b) Use vector methods to find the exact distance between the two particles when $t = 4$.

$\overrightarrow{AB}(4) = \underset{\sim}{r}_B(4) - \underset{\sim}{r}_A(4)$

$= \begin{pmatrix} 13 \\ 10 \end{pmatrix} - \begin{pmatrix} 11 \\ 20 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ -10 \end{pmatrix} \quad (1)$

$\frac{}{6}$

So, distance b/n A + B when $t=4$ $= \left|\begin{pmatrix} 2 \\ 10 \end{pmatrix}\right|$

$= \sqrt{104}$ or $2\sqrt{26}$ (1)

(c) Use vector methods to prove that the particles collide and find the time and position vector of the point of collision.

$\underset{\sim}{i}$ components equal when

$3t - 1 = 2t + 5$

$t = 6 \quad (1)$

$\underset{\sim}{j}$ components equal when

$5t = t^2 - 6$

$t^2 - 5t - 6 = 0$

$(t-6)(t+1) = 0$

$t = 6$ or $t = 1$

(1) justification of collision

So, A and B collide when $t = 6$

$\underset{\sim}{r}_A(6) = \underset{\sim}{r}_B(6) = \begin{pmatrix} 17 \\ 30 \end{pmatrix}$

Objects collide at $17\underset{\sim}{i} + 30\underset{\sim}{j}$ (1)

End of Resource Free

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ST HILDA'S
ANGLICAN SCHOOL FOR GIRLS INC.

Total Time: 25 minutes

Total Marks: 21 marks

Specialist Mathematics Units 3&4

Topic Test 2

(Wed, May 11th)

Miscellaneous Formulae Sheet, half an A4 size page of notes and ClassPad Calculators are permitted.

Name: ANSWERS

In this section of the test, you should use your ClassPad calculator. You **must** show appropriate mathematics so that your method is clear. Do not write ClassPad instructions in your method. Write only appropriate mathematical notation.

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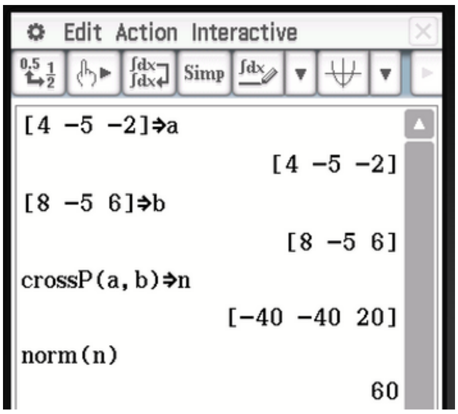
5. [2 marks]

Points A and B have position vectors $4\underset{\sim}{i} - 5\underset{\sim}{j} - 2k$ and $8\underset{\sim}{i} - 5\underset{\sim}{j} + 6\underset{\sim}{k}$ respectively relative to an origin O.
Use vector methods to find the area of ΔAOB.



$$\vec{OA} \times \vec{OB} = \begin{pmatrix} -40 \\ -40 \\ 20 \end{pmatrix}$$

$$|\vec{OA} \times \vec{OB}| = 60 \quad (1)$$

$\overline{2}$

$\therefore$ Area of Parallelogram containing A, O and B $= 60$

So, Area of $\Delta AOB = \frac{1}{2}$ Area of Parallelogram

$= \frac{1}{2} \times 60$

$= 30$ units^2 (1)

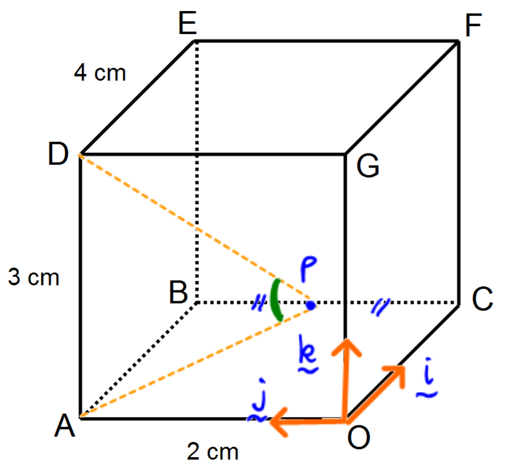
2

6. **[2, 2 & 2 = 6 marks]**

The rectangular prism to the right has base OABC and top of GDEF with G, D, E and F above O, A, B and C respectively.

DE = 4 cm, DA = 3 cm and AO = 2 cm and let the origin be point O.

The coordinates of G relative to the origin are (0, 0, 3)



(a) State the position vector of point

(i) D $\overrightarrow{OD} = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$ (1)

(ii) P, the midpoint of BC $\overrightarrow{OP} = \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix}$ (1)

(b) Find a vector equation of the line through points P and D.

$$\overrightarrow{PD} = \overrightarrow{OD} - \overrightarrow{OP}$$
$$= \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix}$$
$$= \begin{pmatrix} -4 \\ 1 \\ 3 \end{pmatrix} \quad (1)$$

So, eq^n of line through P and D is

$$\underset{\sim}{r}(\lambda) = \overrightarrow{OP} + \lambda(\overrightarrow{PD})$$
$$= \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -4 \\ 1 \\ 3 \end{pmatrix}$$
$$= \begin{pmatrix} 4-4\lambda \\ \lambda+1 \\ 3\lambda \end{pmatrix} \quad (1)$$

or $\underset{\sim}{r}(\lambda) = \overrightarrow{OD} \pm \lambda(\overrightarrow{PD})$
$= \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} \pm \lambda \begin{pmatrix} -4 \\ 1 \\ 3 \end{pmatrix}$

$\overline{6}$
6

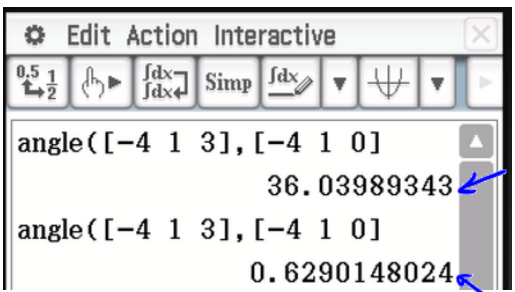
(c) Use vector methods to find the angle the line through P and D makes with the base OABC.

Required Angle $= \angle DPA$

$=$ angle between $\overrightarrow{PD}$ and $\overrightarrow{PA}$ (1)

$=$ angle between $\begin{pmatrix} -4 \\ 1 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} -4 \\ 1 \\ 0 \end{pmatrix}$

$= 36{\cdot}04^\circ$ (2 d.p's) (1) or $0{\cdot}629$ radians



degrees

radians

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7. [1 & 3 = 4 marks]

At 10 am, particle B leaves point P, position vector $2\underset{\sim}{i}+5\underset{\sim}{j}$ metres relative to origin O, with velocity $-\underset{\sim}{i}+5\underset{\sim}{j}$ m/s.

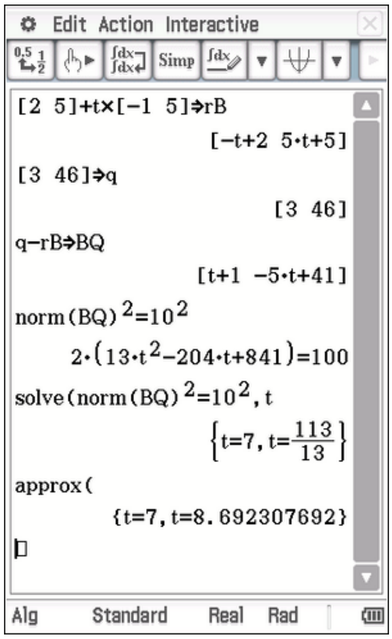
(a) Find the position of B relative to the origin after 5 seconds.

$$\underset{\sim}{r}_B(5) = \begin{pmatrix} 2 \\ 5 \end{pmatrix} + 5\begin{pmatrix} -1 \\ 5 \end{pmatrix}$$

$$= -3\underset{\sim}{i} + 30\underset{\sim}{j} \quad (1)$$

(b) Find the amount of time after 10 am that it takes for B to first be 10 metres from the point Q which has position vector $3\underset{\sim}{i}+46\underset{\sim}{j}$.

$$\underset{\sim}{r}_B(t) = \begin{pmatrix} 2 \\ 5 \end{pmatrix} + t\begin{pmatrix} -1 \\ 5 \end{pmatrix} = \begin{pmatrix} 2-t \\ 5t+5 \end{pmatrix}$$



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$$\overrightarrow{BQ} = \overrightarrow{OQ} - \underset{\sim}{r}_B(t)$$

$$= \begin{pmatrix} 3 \\ 46 \end{pmatrix} - \begin{pmatrix} 2-t \\ 5t+5 \end{pmatrix}$$

$$= \begin{pmatrix} t+1 \\ 41-5t \end{pmatrix} \quad (1)$$

$|\overrightarrow{BQ}| = 10$ when $(t+1)^2 + (41-5t)^2 = 10^2$ (1)

$t = 7$ or $\frac{113}{13} \approx 8.69$

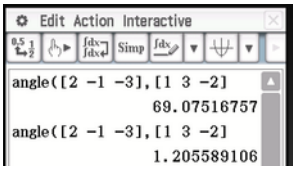
So, B is first 10m from Q 7 seconds after 10am. (1)

4

8. [1 & 3 = 4 marks]

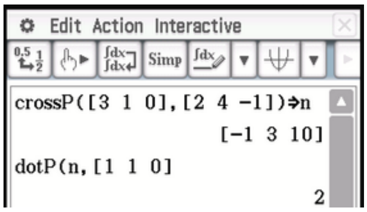
(a) Find the angle between the planes $\underset{\sim}{r}\cdot\left(2\underset{\sim}{i}-\underset{\sim}{j}-3\underset{\sim}{k}\right)=10$ and $x+3y-2z=16$.

Angle between planes = Angle between $\begin{pmatrix}2\\-1\\-3\end{pmatrix}$ and $\begin{pmatrix}1\\3\\-2\end{pmatrix}$



$= 69.075^\circ$ (1) (2dp's) or 1.2056 radians

(b) Find, in scalar product form, the vector equation of the plane $\underset{\sim}{r}=(1+3\lambda+2\mu)\underset{\sim}{i}+(1+\lambda+4\mu)\underset{\sim}{j}-\mu\underset{\sim}{k}$.



$\overline{4}$

$\underset{\sim}{r}=\begin{pmatrix}1\\1\\0\end{pmatrix}+\lambda\begin{pmatrix}3\\1\\0\end{pmatrix}+\mu\begin{pmatrix}2\\4\\-1\end{pmatrix}$

So, the vectors $\begin{pmatrix}3\\1\\0\end{pmatrix}$ and $\begin{pmatrix}2\\4\\-1\end{pmatrix}$ lie in the plane.

$\underset{\sim}{n}=\begin{pmatrix}3\\1\\0\end{pmatrix}\times\begin{pmatrix}2\\4\\-1\end{pmatrix}$

$=\begin{pmatrix}-1\\3\\10\end{pmatrix}$ (2)

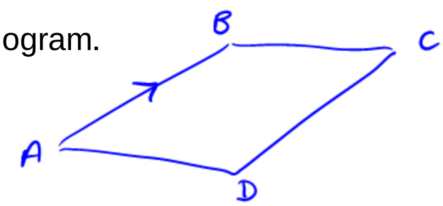
$\therefore$ Equation of Plane is $\underset{\sim}{r}\cdot\begin{pmatrix}-1\\3\\10\end{pmatrix}=\begin{pmatrix}1\\1\\0\end{pmatrix}\cdot\begin{pmatrix}-1\\3\\10\end{pmatrix}$

$\underset{\sim}{r}\cdot\begin{pmatrix}-1\\3\\10\end{pmatrix}=2$ (1)

9. [2 marks]

The points A$(4,-2,3)$, B$(-1,3,4)$, C$(2,4,-2)$ and D$(7,-1,-3)$ form quadrilateral ABCD.

Use vector methods to prove that ABCD is a parallelogram.



$\overline{2}$

$\overrightarrow{AB}=\begin{pmatrix}-1\\3\\4\end{pmatrix}-\begin{pmatrix}4\\-2\\3\end{pmatrix}$

$=\begin{pmatrix}-5\\5\\1\end{pmatrix}$

$\overrightarrow{DC}=\begin{pmatrix}2\\4\\-2\end{pmatrix}-\begin{pmatrix}7\\-1\\-3\end{pmatrix}$

$=\begin{pmatrix}-5\\5\\1\end{pmatrix}$

(1)

$\therefore \overrightarrow{AB}=\overrightarrow{DC}$

Hence, ABCD is a parallelogram

(1) must include reason

6

10. [3 marks]

The position vectors of three non-collinear points A, B and C, with respect to an origin O, are $\underset{\sim}{\mathbf{a}}$, $\underset{\sim}{\mathbf{b}}$ and $\underset{\sim}{\mathbf{c}}$ respectively.

Given that O does not lie in the plane ABC, show that $\alpha+\beta+\gamma=1$ if the point Q with position vector $\underset{\sim}{q}=\alpha\underset{\sim}{\mathbf{a}}+\beta\underset{\sim}{\mathbf{b}}+\gamma\underset{\sim}{\mathbf{c}}$ lies in the plane ABC.

Equation of Plane ABC has equation

$$\underset{\sim}{r} = \vec{OA} + \lambda\vec{AB} + \mu\vec{AC}$$

$$= \underset{\sim}{a} + \lambda(\underset{\sim}{b}-\underset{\sim}{a}) + \mu(\underset{\sim}{c}-\underset{\sim}{a}) \quad (1)$$

$$= (1-\lambda-\mu)\underset{\sim}{a} + \lambda\underset{\sim}{b} + \mu\underset{\sim}{c}$$

As $\underset{\sim}{q}$ lies in the plane

$$\underset{\sim}{q} = (1-\lambda-\mu)\underset{\sim}{a} + \lambda\underset{\sim}{b} + \mu\underset{\sim}{c}$$

So,

$$\alpha\underset{\sim}{a} + \beta\underset{\sim}{b} + \gamma\underset{\sim}{c} = (1-\lambda-\mu)\underset{\sim}{a} + \lambda\underset{\sim}{b} + \mu\underset{\sim}{c} \quad (1)$$

$\frac{}{3}$

$$\Rightarrow \alpha = 1-\lambda-\mu, \quad \beta=\lambda \text{ and } \gamma=\mu$$

So, $\alpha = 1-\beta-\gamma$ (1)

$$\therefore \alpha+\beta+\gamma = 1$$

End of Assessment

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